Exercise 332

For the following problems, consider a restaurant owner who wants to sell T-shirts advertising his brand. He recalls that there is a fixed cost and variable cost, although he does not remember the values. He does know that the T-shirt printing company charges $440 for 20 shirts and $1000 for 100 shirts.

a. Find the inverse function $x = f^{-1}(C)$ and describe the meaning of this function. b. Determine how many shirts the owner can buy if he has $8000 to spend.

Solution

The cost function is linear.

$$C(x) = mx + b$$

x is the number of shirts sold, and m and b are constants to be determined. Use the given information to determine m, the slope.

$$m = \frac{1000 - 440}{100 - 20} = 7$$

The cost function is then

$$C(x) = 7x + b.$$

Use the fact that 20 shirts costs $440 to determine b.

$$440 = 7(20) + b$$

Solve for b.

$$b = 440 - 140 = 300$$

Therefore, the cost function is

$$C(x) = 7x + 300.$$

To get the inverse function, replace x with y, and replace $C(x)$ with x in the equation.

$$x = 7y + 300.$$

Solve for y.

$$7y = x - 300$$

$$y = \frac{1}{7}(x - 300)$$

This is the inverse function: For a given cost x, this function gives the number of shirts that the owner can get. If the owner has $8000 to spend, then

$$y = \frac{1}{7}(8000 - 300) = 1100$$

the owner can get 1100 shirts.